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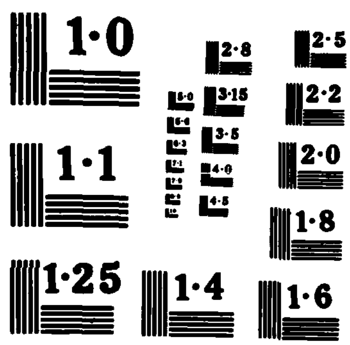

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A SURVEY OF SOME RECENT RESULTS IN CONTINUOUS UNIVARIATE DISTRIBUTIONS

(prepared for presentation at the Centenary Session of the International Statistical Institute, Amsterdam, Netherlands, August 12-22, 1985)

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ABSTRACT:

A description of a survey of developments in continuous univariate distributions - theory and applications - since 1970. As an example, a section on inverse Gaussian distribution (and generalized inverse Gaussian distribution) is appended.

Key Words and Phrases: Continuous univariate distributions, Inference, Generalized inverse Gaussian distribution.

1. INTRODUCTION

Pursuant to our work on advances in discrete distributions (Johnson and Kotz (1982)) we have commenced work on advances in continuous univariate distributions from 1971 onwards. In contrast to the situation for discrete distributions, this field was markedly more fully developed by 1970, in regard to both variety of available distributions and sophistication of methods for fitting. Possibly as a consequence, there is now available a number of very useful survey articles for particular families of distributions. We will make reference to these at appropriate points, indicating the nature of their contents. The interested reader is recommended to study these sources for more precise information.

While taking advantage of the existence of these valuable summaries to reduce the volume of our own work, we believe we should include enough descriptive material to provide an adequate picture of the current situation. In our discussion of distributions already in use before 1971, our discussion will be mostly of methods of fitting (including the closely related topic of estimation of parameters), though there will often be examples of new applications and occasionally some new properties to report. In regard to distributions which have come into use since 1971 we will, in addition, endeavor to provide some indication of the genesis of the distribution, its properties and relations to other distributions.

2. Some General Observations

Two general methods of estimation have appeared especially aimed at fitting distributions with a 'threshold value' - that is a value (unknown) below which the density function is zero, but above which this is not so - such as Weibull, exponential, inverse Gaussian.

(a) Cohen & Whitten (1982, 1985) have suggested modification of maximum likelihood estimation by replacing the equation putting the derivative of likelihood with respect to the threshold parameter equal to zero by one equating the observed and expected values of the first order statistic (the least observation). They also suggest a similar modification to the method of moments, wherein the equation relating third sample and population moments is replaced. The modified method gives results comparable with full *maximum likelihood in the first case,* and often results in substantial improvement in the second.

(b) Cheng and Amin (1983) have proposed a method of maximum product of spacings (MPS), in which the product of the 'spacings' $\Pr[X \le X_{(i)} | \underset{\sim}{\theta}] - \Pr[X \le X_{(i-1)} | \underset{\sim}{\theta}]$ $(i=1,2,\ldots,n+1)$ is maximized with respect to θ. ($X_{(1)} \le X_{(2)} \le \cdots \le X_{(m)}$ are order statistics for a random sample of size n; $X_{(0)} = -\infty, X_{(n+1)} = +\infty$.) This method can be used for any continuous univariate distribution, but is particularly useful or distributions with threshold value(s). The authors claim that

(i) MPS estimation gives consistent estimators under more general conditions than maximum likelihood estimation.

(ii) MPS estimators are asymptotically normal and asympotically as efficient as maximum likelihood estimators when these exist. Ranneby (1984) gives a detailed theoretical treatment of MPS estimation.

A noteworthy feature of recent years has been the attention devoted to simulation techniques, in particular in assessing properties of estimators. One might think that, since any non-singular continuous distribution can be derived from any other by an appropriate monotonic transformation of the variable, it would suffice to have a method for generating values corresponding to a single distribution - for example, a uniform distribution, which would be closely approximated by values from standard tables of random numbers. It is true that some programs do, in effect, use such an approach, but others are more sophisticated and exploit specific features of particular distributions.

There have been some studies comparing different distributions. These are necessarily of a somewhat empirical nature. This is by no means a empirical term - the use of criteria such as average absolute density difference is not often informative for practical purposes. When there is really close mimicry among different distributions, there is very strong inducement to use the one leading to simpler calculations. If, in fact, use of one, rather than another, of two closely-agreeing distributions, makes any substantial difference in an inference procedure, it is the procedure itself (or associated theory) which should be regarded as of doubtful utility. On the more theoretical side there has been work on conditions under which distributions are 'comparable' (e.g. Lisek (1978))

but this seems to hinge on assumptions that one 'ought' to be able to summarize comparisons in terms of a few indices (usually one).

The above remarks should not be taken as disparagement of construction and investigation of new distributions. Indeed, the fact that it is often the case that it is found that a rather complicated distribution, derived from a carefully constructed model, can be adequately represented by a much simpler distribution is the basis for substantial advances in applicability. But the more complicated distribution has to be studied <u>first</u> in order to establish whether there <u>is</u> a close relationship. Also, from a general theoretical point of view, studies of <u>systems</u> of distributions can assist greatly in attaining a broad grasp of the subject.

Space does not permit presentation of even one example of the detailed treatment we wish to use. Section 3, appended here, is such an example, showing a discussion of inverse Gaussian distribution and including discussion of <u>generalized</u> inverse Gaussian distributions.

APPENDIX

3. INVERSE GAUSSIAN (IG) DISTRIBUTION

There are several 'standard' forms for this distribution. We will use the formula

$$f_X(x|\mu,\theta) = \left(\frac{\theta}{2\pi x^3}\right)^{\frac{1}{2}} \exp\left\{-\frac{\theta(x-\mu)^2}{2\mu^2 x}\right\} \tag{1}$$

$$(x > 0;\ \mu,\ \theta > 0)$$

for the PDF. The corresponding CDF is

$$F_X(x|\mu,\theta) = \Phi\left(\frac{(x-\mu)\sqrt{\theta}}{\theta\sqrt{x}}\right) + e^{2\theta/\mu}\,\Phi\left(-\frac{(x+\mu)\sqrt{\theta}}{\theta\sqrt{x}}\right) \qquad (2)$$

where $\Phi(.)$ is the standard normal cumulative distribution function. We will use the notation $IG(\mu,\theta)$ or just IG, for this distribution.

The mean and variance are

$$E[X] = \mu;\ \mathrm{var}(X) = \mu^3/\theta$$

and the skewness is $\alpha_3 = 3\sqrt{(\theta/\mu)}$.

Since taking $\mu=0$ (to make $E[X] = 0$) also makes $\mathrm{var}(X) = 0$, a standardized form of the distribution has to be obtained circuitously. By taking the variable

$$Y = X + \xi \qquad (\xi < 0)$$

so that $E[Y] = \mu + \xi$; $\mathrm{var}(Y) = \mu^3/\theta$; $\alpha_3(Y) = 3\sqrt{(\theta/\mu)}$ and then taking $\mu = -\xi$, $\theta = -\xi^3$ we have

$$E[Y] = 0;\ \mathrm{var}(Y) = 1;\ \alpha_3(Y) = 3|\xi|\ .$$

The CDF of Y is

$$F_Y(y|\alpha_3) = \Phi(y(1+\tfrac{1}{3}\alpha_3)^{-\frac{1}{2}}) + \exp(18\alpha_3^{-2})\,\Phi\left[-(y+6\alpha_3^{-1})(1+\tfrac{1}{3}\alpha_3)^{-1}\right] \qquad (3)$$

Chan et al. (1983) give tables of $F(y|\alpha_3)$ to 6 decimal places for $y = -3.0(0.1)5.9$ with $\alpha_3 = 0.0(0.1)1.2$ and $y = -1.5(0.1)7.4$ with $\alpha_3 = 1.3(0.1)2.5$.

The distribution of Y is a 3-parameter inverse Gaussian (parameters μ, θ, ξ); a 1-parameter inverse Gaussian obtained by taking $\theta = \mu^2$ and $\xi = 0$ has been discussed by Vodă (1973) (See also Iliescu and Vodă (1977)). It has PDF

$$f_X(x|\mu) = \mu(2\pi x^3)^{-\frac{1}{2}} \exp\{-\tfrac{1}{2} x^{-1}(x-\mu)^2\} \tag{4}$$

An excellent summary of properties of the inverse Gaussian distribution, established prior to 1978, is provided in Folks and Chhikara (1978).

An important generalization of the inverse Gaussian distribution, introduced by Good (1953) and studied by Wise (1971, 1975) has attracted considerable attention recently, exemplified especially by an exhaustive and interesting monograph by Jørgensen (1982). He uses an additional parameter, λ, and defines the PDF as

$$\frac{(\psi/\chi)^{\frac{1}{2}\lambda}}{2\,K_\lambda(\sqrt{(\psi\chi)})} x^{\lambda-1} \exp\{-\tfrac{1}{2}(\chi x^{-1} + \psi x)\} \qquad (x > 0) \tag{5}$$

where $K_\lambda(.)$ is a modified Bessel function of the third kind. If $\lambda = -\frac{1}{2}$ we have an inverse Gaussian distribution with parameters $\theta = \chi$; $\mu = \sqrt{(\psi/\chi)}$. Other special cases are

$\lambda = \frac{1}{2}$ - distribution of the reciprocal of an inverse Gaussian variate.

$\chi = 0$, $\lambda > 0$ - gamma distribution.

and $\psi = 0$, $\lambda < 0$ - reciprocal gamma distribution.

For $\lambda = 0$ the 'hyperbolic distribution' (Barndorff-Nielsen (1978), Rukhin (1974)) is obtained.

An alternative form of the PDF, more symmetrical in appearance, is used by Jørgensen (1982). This is obtained by putting $\omega = \sqrt{\chi\psi}$; $\eta = \sqrt{(\chi/\psi)}$ leading to

$$f_X(x) = \{2\eta^{\lambda} K_{\lambda}(\omega)\}^{-1} x^{\lambda-1} \exp\{-\tfrac{1}{2}\omega(\eta^{-1}x + \eta x^{-1})\} \tag{6}$$

$$(x > 0;\ \omega,\ \eta > 0)$$

If $\chi = 0$ or $\psi = 0$, so that $\omega = 0$ the IG distribution (1) corresponds to (6) with $\lambda = -\frac{1}{2}$, $\omega = \theta/\mu$, $\eta = \mu$ (and $K_{-\frac{1}{2}}(\omega) = \sqrt{(\pi/2)\omega^{-\frac{1}{2}}e^{-\omega}}$).

<u>Hazard</u> <u>Function</u>. For $\lambda < 1$ the hazard function is unimodal with zero initial value and asymptotic value $\psi/2$. The mode m_h of the hazard function satisfies

$$\chi(2(1-\lambda))^{-1} \leq m_h \leq \chi(1-\lambda)^{-1}$$

For $\chi = 0$ we have a Gamma distribution with $0 < \lambda < 1$ which is a degenerate case with the hazard function infinite at the origin and <u>decreases</u> towards the asymptotic value $\psi/2$. Thus for the reciprocal gamma distribution ($\psi = 0$, $\lambda < 0$) the hazard function has zero asymptotic value.

For $\lambda \geq 1$ the hazard function starts with zero and <u>increases</u> towards the asymptotic value $\psi/2$. The exception is the exponential case ($\chi = 0$, $\lambda = 1$) when the hazard rate is constant and equals $\psi/2$. (cf. Jørgensen (1982)).

Approximations

Whitmore and Yalovsky (1978) have proposed an approximation to the IG distribution based on supposing

$$\tfrac{1}{2}\,\omega^{-\frac{1}{2}} + \omega^{\frac{1}{2}} \log(X/\eta) = \tfrac{1}{2}\sqrt{\frac{\mu}{\theta}} + \sqrt{\frac{\theta}{\mu}}\,\log(X/\mu) \qquad (8)$$

to have a standard normal distribution.

Jørgensen (1982) reports an approximation based on supposing $-\lambda^{3/2}\chi^{-1}(2X + \lambda^{-1}\chi)$ to have a standard normal distribution, and suggests that even better approximation might be obtained by using log X as approximately normal.

Estimation Estimation of the 3-parameter inverse Gaussian distribution with PDF

$$f_Y(y|\mu,\theta,\ ) = \left\{\frac{\theta}{2\pi\,(y-\xi)^3}\right\}^{\frac{1}{2}} \exp\left\{\frac{\theta(y-\xi-\mu)^2}{2(y-\xi)\mu^2}\right\} \qquad (9)$$

$$(y > \xi\,;\ \theta,\mu > 0)$$

has been discussed by Padgett and Wei (1979), Cheng and Amin (1981), Jones and Cheng (1984), Chan et al. (1984), and Cohen and Whitten (1985).

Moment and maximum likelihood estimation is discussed by Padgett and Wei; and maximum likelihood estimation by Cheng and Amin, with special

attention to consistency and asymptotic efficiency. The two methods of estimation are compared by Jones and Cheng, who find maximum likelihood to be clearly superior. Modified maximum likelihood and moment estimation, in which the equation obtained by equating expected and observed values of the first (least) order statistic replaces the maximum likelihood equation equating $\partial \log$ (likelihood) / $\partial \xi$ to zero, or that equating sample and population values of the third moment to zero, are discussed by Chan et al. Cohen and Whitten provide tables to assist in computation of the solution of the newly introduced equation for ξ in terms of μ and θ.

Maximum likelihood estimation of the 2-parameter IG distribution is discussed by Gupta (1973). Bayesian estimation of reliability was considered by Padgett (1981), which was recently generalized by Howlader (1985).

The information matrix for a mixture of two IG distributions has been evaluated by Al-Hussaini and Ahmad (1984).

<u>Genesis</u> Huff (1975) has given a heuristic derivation of the well-known result that the inverse Gaussian can arise as the distribution of first passage time in Brownian motion. Barndorff-Nielsen et al. (1978) have shown that generalized inverse Gaussian distribution with $\lambda < 0$ can arise as the distribution of first passage time of a time homogeneous diffusion process $x(t)$ over $[0,\infty)$ with infinitesimal mean $\beta(x)$ and variance $\alpha(x)$, where

$$\beta(x) = -\sqrt{\alpha(x)}\left\{\frac{2\lambda-1}{2\theta(x)} + \frac{\sqrt{\psi}\;K_{\lambda-1}\,(\theta(x)\sqrt{\psi})}{K_{\lambda}(\theta(x)\sqrt{\psi})}\right\} + \frac{1}{4}\frac{d\,\alpha(x)}{d\,x} \quad (\text{if } \psi > 0)$$

$$\beta(x) = \sqrt{\alpha(x)} \;.\; \frac{2\lambda+1}{2\,\theta(x)} + \frac{1}{4}\frac{d\,\alpha(x)}{d\,x} \qquad (\text{if } \psi = 0)$$

The parameters of the GIG distribution are λ, $\theta^2(x(0))$ and ψ. where Jørgensen (1982) shows that the GIG can be regarded as a limiting case of generalized hyperbolic distribution.

Applications Marcus (MS 1974-5) has suggested that the IG distribution be used in place of the lognormal distribution when longtailed distributions are expected to be appropriate - for example in distribution of sizes of particles in aggregrates (Bardsley (1980)). If distributions of sums of individual random variables (convolutions) are to be studied, it may be possible to use the fact that if IG variables $X_i \frown IG(\mu_i, \theta_i)$ have the same value ϕ, say, for the ratio of variance to expected value (μ_i^2/θ_i) then

$$(X_1 + X_2 + \ldots + X_n) \text{ has an IG } \left(\sum_{i=1}^{n} \mu_i,\ \phi^{-1}\left(\sum_{i=1}^{n} \mu_i\right)^2\right) \text{ distribution.}$$

IG distributions have been used to represent duration of strikes (Lancaster (1972)); hospital stays (Whitmore (1975, 1976)); labor turnover (Whitmore (1979)); aggregate insurance claims (Seal (1978)); lifetime (Chhikara and Folks (1977); and sentence length (Sichel (1974, 1975)). A critical conparison of the use of lognormal and IG distributions for duration of strikes has been given by Lawrence (1984).

An application in accelerated life testing has been described by Bhattacharyya and Fries (1982). It is supposed that for a given stress, x, the conditional distribution of lifetime is $IG((\alpha + \beta x)^{-1}, \lambda)$.

<u>Inference</u> The statistic $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^{-1}\right)$ is minimal sufficient for the IG distribution; $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^{-1}, \sum_{i=1}^{n} \log X_i\right)$ is minimal sufficient for the GIG distribution. Jørgensen (1982) uses the notation $X_{.}$, X_{-1}, $X_{\sim}$ for the three last quantities, and we will follow this example, also introducing $\bar{X} = n^{-1} X_{.}$, $\bar{X}_{-1} = n^{-1} X_{-1}$; $\bar{X}_{\sim} = n^{-1} X_{\sim}$.

The well-known analogy between analysis of data based on normal and on IG parent distributions, (clearly presented by Folks and Chhikara (1978)), whereby the ML estimators of μ and θ for IG data are

$$\hat{\mu} = \bar{X}_{.}\,;\ \hat{\theta} = (\bar{X}_{\sim} - \bar{X}_{.}^{-1})^{-1},$$

$\hat{\mu}$ and $\hat{\theta}$ are mutually independent, $\hat{\mu}$ has a IG(μ,$n\theta$) distribution and $n\,\theta/\hat{\theta}$ has a χ^2_{n-1} distribution, does not extend to generalized IG distributions. Folks and Chhikara (1978) draw attention to the algebraic formulae

$$\tfrac{1}{2}\,\theta\,\mu^{-2} \sum_{i=1}^{n} X_i^{-1}(X_i-\mu)^2 = \tfrac{1}{2}\,\theta \sum_{i=1}^{n} (X_i^{-1} - \bar{X}^{-1}) + \tfrac{1}{2}\,n\theta\,\mu^{-2}\,\bar{X}^{-1}(\bar{X}-\mu)^2$$

and

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} \left(X_{ij}^{-1} - \bar{X}_{..}^{-1}\right) = \sum_{i=1}^{k} n_i(\bar{X}_{i.}^{-1} - \bar{X}_{..}^{-1})$$

$$+ \sum_{i=1}^{k} \sum_{j=1}^{n_i} (X_{ij}^{-1} - \bar{X}_{i.}^{-1})$$

where $\bar{X}_{i.} = n_i^{-1} \sum_{j=1}^{n_i} X_{ij}$ and $\bar{X}_{..} = (\sum_{i=1}^{k} n_i)^{-1} \sum_{i=1}^{k} n_i \bar{X}_i$. These are analogs, for IG parent populations, to the standard decompositions of sums of squares, used in analysis of variance for data in one-way classification by groups, for normal parent populations.

They also show that for the analog of a standardized normal variable

$$U = (n\theta)^{\frac{1}{2}}(\bar{X}-\mu)(\mu\bar{X}^{\frac{1}{2}})^{-1}$$

U^2 is distributed as χ^2 with 1 degree of freedom but U is not normally distributed. Similarly, defining

$$W = U/\{\frac{n}{n-1} \cdot \theta/\hat{\theta}\}^{\frac{1}{2}}$$

W^2 is distributed as the square of Student's t with (n-1) degrees of freedom, but W is not distributed as Student's t.

Folks and Chhikara (1978) also provide results relevant to comparison of populations with IG distributions and to regression analysis.

Uniform minimum variance estimation of parameters of IG distributions is discussed by Iwase and Seto (1983).

<u>Recent papers on GIG Distributions</u>

Letec and Seshadri (1983) have provided a characterization of GIG distributions.

Embrechts (1983) has shown that the GIG distribution (5) belongs to a class of subexponential distributions.

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